# the Water in the Soil – Part 3

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While working on this approach to try and figure out where pore water pressure comes from, and how it might be calculated, the realization that hydrodynamics is a necessary part of Soil Mechanics became increasingly obvious. In hindsight I'm surprised it took me so long to come to appreciate how crucial a partner Fluid Mechanics is of Soil Mechanics, especially when it should have been clear from the start that deformation of a saturated soil-structure is basically a matter of water moving around obstructions.

In the last article I used the Coefficient of Drag [$C_D$] to calculate the forces acting on a solid as it moved through water. This is of course a term borrowed from our hydraulic colleagues. Now in this article I want to transform that term into our own language and rules of behaviour.

But first, in order to be able to work effectively with hydrodynamics it is necessary to become reacquainted with another of their key terms without which it would be all but impossible to advance.

### a Word about the Reynolds Number

I've tried to keep away from this parameter which for most practicing geotechs will be all but a silent echo from student days. But the Reynolds Number [$R_E$] is too useful to be done without. And there is no good substitute to replace it. Once I found the need to invoke the ideas and tools of hydrodynamics I knew I had to learn to live with the Reynolds Number too.

Back in 1938 Hunter Rouse published a curve relating $C_D$ values for rough spheres to $R_E$. His curve is reproduced here as the grey line in Figure 7. As you can see it covers the full range of practical interest to us.

Fortunately, as it turns out, for geotechnical purposes the value of $R_E$ is given to an accuracy of two decimal places by the simple multiple:

$$R_E = D\ v$$

provided the diameter "D" is in millimetres, the relative velocity "v" is in millimetres per second, and the water temperature is about 20° C. These conditions result in the combination of the other hydraulic parameters becoming equal to unity.

### the Problem with Drag

The idea of simply adopting $C_D$ wholesale made me a bit nervous. Nervous, mainly, because I had no feel for it. It's not something I'd ever used in the field, like cohesion or friction - a parameter I could pull out of my head for a back-of-the-envelope estimate on the run. At liquefaction velocities this parameter can vary from 3,000,000 for fine silt, to less than 0.4 for gravel - and for no good reason intuitively apparent to me.

I felt the need to find some way of relating to the basic physics behind this widely (I might say wildly) varying parameter. And preferably, if it were ever to be confidently adopted in practice, then in a geotechnically analogous way. As it turned out, there are two geotechnical mechanisms with which we are all familiar and which can be used to get quite close to replicating this very useful, but rather intimidating parameter. The two geotechnical analogies I found that fitted the bill were bearing capacity of foundations, and standpipe piezometers.

### Bearing Capacity

Anyone who dived into the water from a bit too high up doesn't need to be told that water resists penetration. It's all a matter of speed of entry. This is because water is viscous and therefore its resistance to penetration increases with the rate at which it has to deform. So the thought arose that the interaction of a particle moving in water might be equivalent to steady state bearing capacity displacement, where the strength of the "foundation" was proportional to the rate of straining.

So lets consider that the Drag Force on a sphere, or maybe just some significant fraction of it which I'll call the Bearing component [$F_B$], is simply equal to the ultimate bearing capacity [$q_{ult}$] of a circular footing of the same size. In normal terminology this is:

$$F_B = q_{ult} A = c N_c A$$

where:

| | |
|---|---|
| c | shear strength |
| $N_c$ | bearing capacity factor |
| A | equatorial area of sphere. |

To make this work two shear strength terms need to be related across the disciplines: How could soil shear strength "c" be expressed in terms of water viscosity " μ" ? Both these terms are defined as resistance to shearing force, it is only that the latter is also directly dependent on the rate of straining, and consequently has stress-time [Pa.s] units. To sort this out requires a bit of mathematical juggling.

***Dimensional Analysis***

In order to make viscosity dimensionally equivalent to cohesion it needs to be multiplied by one or more parameters, which taken together, have the dimension $\text{Time}^{-1}$. Velocity [m/s] suggests itself as a candidate in this situation, and would, if divided by some significant length "Y" [m], resolve the incompatibility satisfactorily. Therefore, according to dimensional analysis theory, the following equation, where "sv" is some significant velocity, and "Y" is some significant linear dimension, must hold true:

$$c = \mu \, sv / Y$$

[Pa] = [Pa . s . m/s . 1/m] = [Pa]

Consequently, we may now write:

$$F_B = \mu \, sv \, ( N_c / Y ) \, A$$

To try to discover what "Y" might be, I did a regression analysis directly comparing this $F_B$ component of Drag Force with the standard equation $F_D$ as given in Part 2. The result was pleasantly surprising. Over a large range of the smaller sphere sizes and lower relative velocities there was complete agreement between the two formulations for Drag Force once "$N_c$ / Y" was given the value "12 / diameter" and the velocity "sv" was simply the "v" term representing relative motion between the phases.

So how to make geotechnical sense of $N_c$ / Y = 12 / diameter?

We know that for a circular footing $N_c$ has a value between 5.7 and 6.2 when shape factors are included, therefore setting $N_c$ = 6 seems acceptable. And doing that would mean Y = spherical diameter / 2, or simply, the radius of the sphere. Therefore, we can now express the equivalent cohesion in the bearing capacity analogy in terms of water viscosity as follows:

$$c = \mu \, v / \text{radius} = 2 \, \mu \, v / D$$

And it follows that the Bearing component of Drag Force becomes:

$$\begin{aligned} F_B &= q_{ult} A = c N_c A \\ &= ( 12 \, \mu \, v / D ) \, A \end{aligned}$$

What this amounts to is that if this component [$F_B$] were the full equivalent of the Drag Force [$F_D$] then $C_D$ would equal 24 μ / ρ v D. This equivalent value is plotted as the solid red line marked "B" on Figure 7.

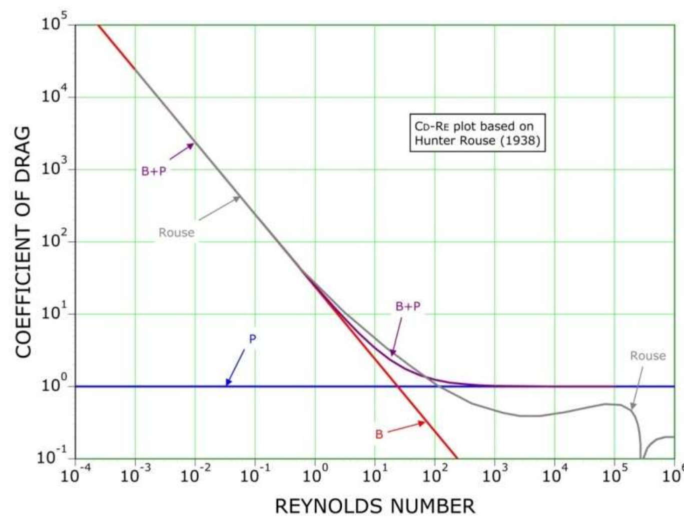


*Figure 7 : The coefficient of drag*

**Standpipe Piezometer**

At any point where flowing groundwater is locally blocked, and made to become stationary, the kinetic energy of the water is transformed into hydraulic pressure. The energy conversion from dynamic to static is given by:

$$h = v^2 / 2 g$$

where "h" is the pressure head in metres of water. It represents the additional amount by which the water level in a standpipe piezometer will be increased by being placed within flowing water, as opposed to stationary water. This is because the water level in the standpipe, being static and balanced, must equal the energy level of the flowing water to which it is exposed. The incremental water pressure associated with this condition is obtained by multiplying the head by the unit weight of water [$\gamma_w = \rho g$].

Similarly to what I did above with the Bearing component I now want to consider that the Drag Force, or some fraction of it which I'll call the Pressure component [$F_P$], is equal to that part of piezometric pressure derived from the velocity head, so that:

$$F_P = ( \gamma_w h ) A$$
$$= ( \rho g v^2 / 2 g ) A$$
$$= ( \rho v^2 / 2 ) A$$

If this component [$F_P$] were the full equivalent of the Drag Force [$F_D$] then $C_D$ would equal 1. That is why the blue line marked "P" on Figure 7 is plotted horizontally through unity.

**a new term: the L–factor**

As noted above, the grey line in Figure 7 is the relationship between $C_D$ and $R_E$ determined by Hunter Rouse back in 1938. What I want to do now is show that this unfamiliar parameter $C_D$ can be replaced with a simple combination of the two geotechnical pressure terms: $\gamma_w$ h, and $q_{ult}$.

What we can see/learn from the red line marked "B" on Figure 7 is that setting $C_D$ equal to 24 $\mu$ / $\rho$ v D, the value from which $F_B$ is derived, this swap provides an exact replication of $C_D$ for any value above about 30 or 40; but thereafter, it is an inaccurate underestimation. This means that replacing the original Drag Force $F_D$ by the Bearing component $F_B$ is a reasonable thing to do for $R_E$ values up to about 1.

The blue line marked "P" running along the $C_D$ = 1 ordinate in this figure corresponds to assuming $F_D$ could be replaced by the Pressure component equation, $F_P$, that is assuming $F_D = F_P$. It is obvious that this effort at replication leads to gross underestimations until it cuts the $C_D$ curve at about $R_E$ = 100. Beyond this point it produces equivalent $C_D$ values which are an overestimation by a factor of about 2.

Therefore, the adoption of neither analogy on which these lines are based is acceptable in its own right for the full range of $C_D$ of interest to us.

A purple line (with open circles) "B + P" shows the result of the simple addition of the ordinates of lines B and P: This is equivalent to assuming that the effects of both the Bearing and Pressure components act simultaneously on the particle. By this means the departure from the experimental curve is reduced considerably, to an amount which, in the context of a parameter which has a practical variability of seven orders of magnitude, might be considered a reasonably approximation. Nevertheless, because I want to bring the combined influences of the two components ($F_B$ and $F_P$) into full alignment with Rouse's $C_D$ over the full range of $R_E$ it became necessary to introduce and apply a correction factor. This is the " L–factor ".

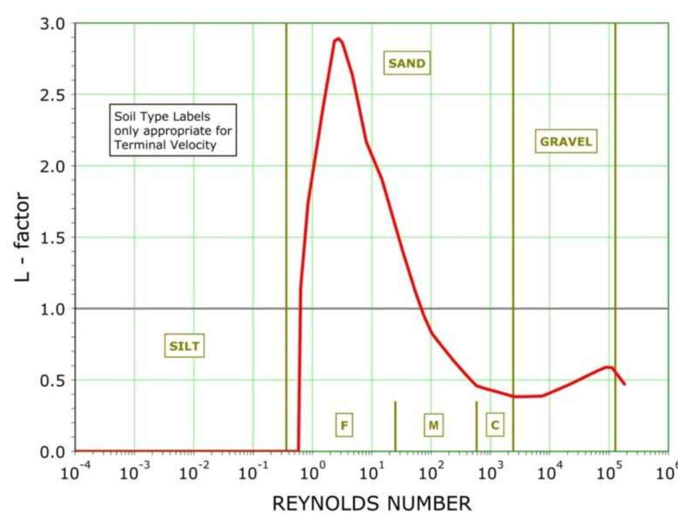


*Figure 8 : The L-factor*

In applying an alignment factor to the two separate and additive components of Drag there is a choice. Without resulting in any inaccuracy to the value of the Drag Force $F_D$ computed, a non-dimensional L-factor can either be applied as an overall multiplier, as in L ( $F_B + F_P$ ), or as a component-specific multiplier, as in ( $F_B$ + L $F_P$ ). At this stage of the development it is more instructive to use the latter alternative, and so, in Figure 8 the appropriate values of the L-factor are plotted for use in the equation:

$F_D = F_B + L\, F_P$

Across the range of interest to us the values of the L-factor vary between 0.0 and 2.9. Here then are the sort of numbers I can keep in my head, something I could never do with $C_D$, the equivalent values of which vary between 0.39 and 3,350,000 over the same domain.

You will see "soil-type" labels marked across the $R_E$ range in Figure 8. It is necessary to say that these labels apply only at Terminal Velocity. But because up till now we have concerned ourselves mainly with the liquefaction phenomenon I have added them to help put thing into some context.

**Modifying Mechanics – from Fluid to Soil**

At this stage we can now rewrite the hydraulics style formula for Drag Force, $F_D = C_D\, \rho\, A\, v^2 / 2$, in geotechnical terms as follows:

$F_D = F_B + L\, F_P$

where:

$F_B = q_{ult}\, A$

$F_P = \gamma_w\, h\, A$

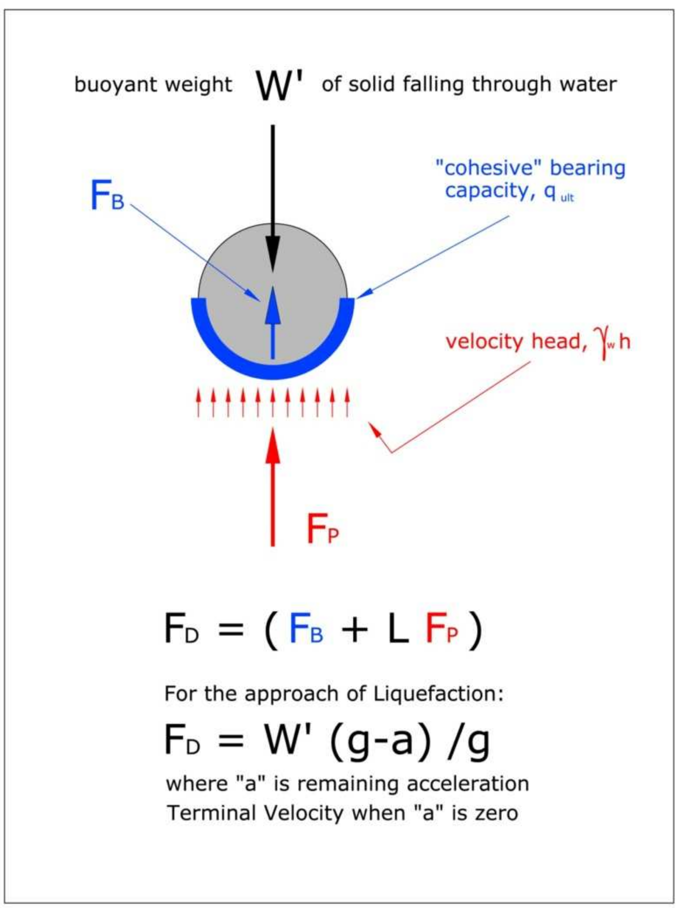


*Figure 9 : Forces on falling ball*

I've drawn the free-body diagram in Figure 9 to help illustrate the balance of forces involved in this approach. This shows the formulation for the special case of liquefaction. Later in this series of articles the more general case of soil-structure deformation will be illustrated.

This geotechnical version, which gives exactly the same answers as the original, allows contemplation of solid-to-water interaction in terms of two separate mechanisms with which we are quite familiar ourselves. And now we are free to think of fine particles as gradually settling footings, and to think of gravel as solid impediments confronting the impulse of flowing water.

But there is more to the above than just appropriation of the good work of our hydraulics colleagues. What we might now have at our disposal is a two-part elemental vector pointing along a potential gradient parallel with the thrust of soil-structure distortion. This comes about because the force $F_D$ cited above is generated by each individual particle in that part of the saturated mass which is being moved. It is quite similar to a seepage gradient where water moves through soil under the influence of an external hydraulic gradient. The difference is that in the case of steady state seepage there is no instability or geometric alteration of the soil-structure, whereas what we are dealing with in these articles is pore pressure change brought about by a deforming soil-structures.

In practical terms I find it interesting that for relative velocities around those associated with liquefaction, the L-factor has the following values: across the full silt size range L equals zero; it reaches a peak for fine sands; and then falls to around 0.5 for gravels where turbulent flows are to be expected.

It is a consequence of how the Bearing component was formulated that it may be concluded that the term $F_B$ is not a contributor to pore pressure. Herein, the energy derived from the work done as the Drag Force progresses may be spent entirely in overcoming viscosity, or following the analogy adopted here, "cohesion" and, I suppose, just dissipated as entropy/heat. Similarly, it is consistent to presume that it is only the Pressure component $F_P$ which contributes to pore water pressurization, and this takes place as kinetic energy is converted to static potential on the upstream side of solids confronting the water's relative velocity. Following this line of reasoning we will proceed from here on the understanding that all to do with excess pore water pressure in soils under deformation is contained in the term $F_P$, and that $F_B$ is a thing apart.

**Glossary of Terms**

$$F_D = (C_D \rho v^2/2) A$$

$$F_B = (12 \mu v / D) A = c N_C A = q_{ult} A$$

$$F_P = (\rho v^2 / 2) A = \gamma_w h A$$

where:

| | | |
|---|---|---|
| $F_D$ | Drag Force | N |
| $C_D$ | Coefficient of Drag | - |
| $\rho$ | mass density of water | $kg/m^3$ |
| v | relative velocity | m/s |
| A | equatorial area of sphere | $m^2$ |
| $F_B$ | Bearing component | N |
| $\mu$ | viscosity of water | Pa.s |
| D | diameter of sphere | m |
| c | cohesion | Pa |
| $N_C$ | bearing capacity factor | - |
| $q_{ult}$ | ultimate bearing capacity | Pa |
| $F_P$ | Pressure component | N |
| $\gamma_w$ | unit weight of water | $N/ m^3$ |
| h | velocity head | m |

**in the Next Article**

Up to this point I've been looking at the behaviour of a single particle falling through water because that, to a large degree, is what I think liquefaction is all about. What I want to do in the next article is to generally conclude my thoughts on this particular type of failure. Also at that time I will suggest: why silts are not as prone to liquefaction as sands seem to be; point out the areas of agreement and conflict between this proposal and the triaxial testing at Harvard and UBC; and, discuss the comparative effects of earthquake shear waves and surface waves on a saturated soil-structure.